



MAX PLANCK INSTITUTE  
FOR DYNAMICS OF COMPLEX  
TECHNICAL SYSTEMS  
MAGDEBURG



COMPUTATIONAL METHODS IN  
SYSTEMS AND CONTROL THEORY

# Introduction to focus group topics

The **f(A)bulous** workshop on matrix functions and exponential integrators

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Supported by:





1. Background
2. Knowledge transfer
3. High-performance and energy-aware computing
4. Benchmark problems and FAIR comparisons
5. Instructions for focus groups



# Why $f(A)b$ ?

- From a mathematical point of view...
  - More challenges due to nonlinearity
  - Rich theories: complex analysis, functional analysis, PDEs, etc.
- From a computer science point of view...
  - Adaptation of tools and methods for  $Ax = b$
  - Nontrivial implementations required for robust solvers
- From an applications point of view...
  - Lattice quantum chromodynamics
  - Evolutionary advection-diffusion-reaction equations
  - Computational fluid dynamics
  - Chemical master equation
  - Gaussian processes
  - Stiff matrix differential equations
  - Network analysis



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- General (dense) matrix functions:  $f(A)$
- Matrix functions times a vector:  $f(A)\mathbf{b}$
- Exponential integrators:  $\exp(tA)\mathbf{b}$
- Related problems:
  - $\text{trace}(f(A))$
  - $\mathbf{b}^T f(A)\mathbf{b}$
  - $f\{A, B\}(C)$ 
    - Lyapunov, Sylvester, and Stein equations
    - Fréchet derivatives
    - $\kappa(f(A))$



<https://commons.wikimedia.org/wiki/>

Category:Elephants



# Key questions

- What do these problems have in common? What are the underlying “kernels”?
- What techniques have been tested for one problem, but not yet another?
- How do these problems differ significantly, i.e., where should we be careful not to over simplify?
- Which functions  $f$  aren't getting enough attention?
- What role could / should machine learning play here?
- How do we measure success / performance / effort / accuracy / efficiency / stability?
- How can we improve communication between diverse members of this community?



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MareNostrum-4 in Barcelona

[https://commons.wikimedia.org/wiki/File:  
2017\\_BSC\\_Superordenador\\_MareNostrum-4\\_  
Barcelona-Supercomputing-Center.jpg](https://commons.wikimedia.org/wiki/File:2017_BSC_Superordenador_MareNostrum-4_Barcelona-Supercomputing-Center.jpg)

- Machine access
- Scaling
- Parallelization
- Fault tolerance
- Hardware limits– CPU speed, bandwidth, latency, RAM, harddrive storage
- Operating costs
- CO2 emissions



# Key questions

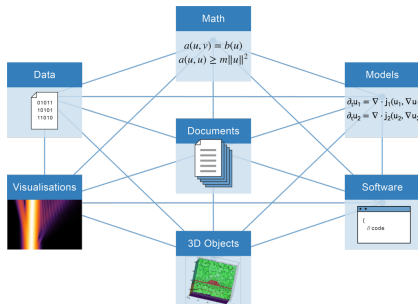
- What strategies can transfer from linear systems? What doesn't transfer?
- What existing tools and libraries can be adapted, built upon, or extended?
- How to optimize the dense problem?
- What should computational models for  $f(A)\mathbf{b}$  look like?
- What challenges do exponential integrators or other specific  $f$  face in particular?
- Is  $f(A)\mathbf{b}$  ready for exascale?
- What role could / should machine learning play here?
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- FAIR principles: Findability, Accessibility, Interoperability, and Reproducibility
- Benchmark problem collections
- Algorithm comparison workflows
- Bare minimum: publish your code and data with an open license!



Borrowed from the MAThematical  
Research Data Initiative (MaRDI)

[www.mardi4nfdi.de](http://www.mardi4nfdi.de)



# Key questions

- What should benchmark problems look like for  $f(A)\mathbf{b}$  and exponential integrators? Are there existing collections that we can adapt? What types of problems would help you in your own research?
- What problems arise when you try to share your data and code? What problems arise when you try to use someone else's data and code?
- How do we incentivize community contributions?
- When developing new algorithms, how do we establish what is state-of-the-art?
- How do we define “algorithm isotopes”?
- What role could / should machine learning play here?
- How can we improve communication between diverse members of this community?





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# Choose your topic...



- ...find your group leader<sup>a</sup>
  - Marcel: Knowledge transfer
  - Stéphane: HPC
  - Kathryn: FAIR Benchmarking
- ...find somewhere nice to work
  - Prigogine (here)
  - Wiener (next door)
  - Technikum (3rd floor with a foosball table)
- ...appoint 1-2 people to take notes
- ...grab some sugar and caffeine
- ...discuss!

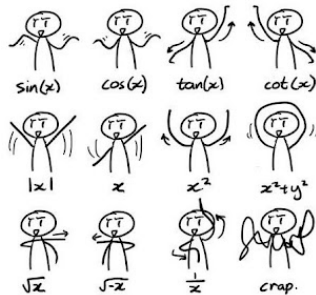
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<sup>a</sup>Try to evenly distribute!



- Plan to present your results tomorrow afternoon
  - 30 min per group
  - Any format welcome:
    - Flipchart
    - Typed notes
    - Powerpoint/Beamer
    - Chalkboard
    - Interpretive dance
- Be creative, controversial, counterintuitive... but courteous!
- Extra motivation: submissions to ETNA special issue due 21 January 2024

## Beautiful Dance Moves





# Where to find these slides



<https://indico3.mpi-magdeburg.mpg.de/e/fabulous2023>