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Projected exponential methods for stiff dynamical low-rank approximation problems

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The numerical integration of stiff equations is a challenging problem that needs to be approached by specialized numerical methods. Exponential integrators [1] form a popular class of such methods since they are provably robust to stiffness and have been successfully applied to a variety of problems. The dynamical low-rank approximation [2] is a recent technique for solving high-dimensional differential equations by means of low-rank approximations. However, the domain is lacking numerical methods for stiff equations since existing methods [3, 4, 5] are either not robust-to-stiffness or have unreasonably large hidden constants.

In this talk, we focus on solving large-scale stiff matrix differential equations with a Sylvester-like structure,

$$\dot{X}(t) = AX(t) + X(t)B + G(t, X(t)), \quad X_0 = X(0),$$

that admit good low-rank approximations. We propose two new methods that have good convergence properties, small memory footprint and that are fast to compute. The theoretical analysis shows that the new methods have order one and two, respectively. We also propose a practical implementation based on Krylov techniques.
The approximation error is analyzed, leading to a priori error bounds and, therefore, a mean for choosing the size of the Krylov space. Numerical experiments are performed on several examples, confirming the theory and showing good speedup in comparison to existing techniques.

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[5] Ostermann, A., Piazzola, C., & Walach, H. (2019). Convergence of a low-rank Lie–Trotter splitting for stiff matrix differential equations. SIAM Journal on Numerical Analysis, 57(4), 1947-1966.

Primary author: CARREL, Benjamin (University of Geneva)

Co-author: Prof. VANDEREYCKEN, Bart (University of Geneva)

Presenter: CARREL, Benjamin (University of Geneva)

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