

$f(A)$ bulous ladder networks: From "rigorous" computation of $b^* \exp(tA)b$ to learning $f(A)b$ with unknown A

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It is well known that the Lanczos tridiagonal matrix can be transformed to an equivalent finite-difference scheme, with the coefficients obtained from the Stieltjes continued fraction. We show the usefulness of such representation on two seemingly unrelated problems.

The first one is "rigorous" computation of the exponential matrix moments $b^* \exp -tAb$. Here we use the finite-difference representation of the Lanczos tridiagonal matrix to compute sharp upper and lower a-posteriori bounds on the solution. These bounds converge strictly monotonically and even remain valid under computer roundoff, when the Lanczos algorithm loses orthogonality, hence the name "rigorous".

The second problem is learning or data-driven computation of $f(A)b$, where A is a s.p.d differential operator with unknown coefficients and f being $\exp(-tA)$, $\cos(\sqrt{A})$, or $(A + sI)^{-1}b$. As the data we use the corresponding SISO transfer function $b^* f(A)b$ or the matrix moments. Such problems are paramount in remote sensing and other "noninvasive problems", e.g., radar imaging and medical applications, where measurements are not available in the interior problem. We first compute the finite-difference representation using the data-driven Lanczos algorithm, and then compute the state solution $f(A)b$ via the corresponding finite-difference problem. Time permitting, we show generalization to multidimensional PDE operators in MIMO and SAR (Synthetic Aperture Radar) setups.

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