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Petrov-Galerkin Krylov methods for algebraic Riccati equations

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Finding the unique stabilizing solution $X = X^H$ of a large-scale continuous-time algebraic Riccati equation (CARE) $0 = R(X) := A^H X + XA + C^H C - XBB^H X$ with a large, sparse n-x-n matrix A, an $n \times m$ matrix B and a $p \times n$ matrix C is of interest in a number of applications. It is assumed, that B and C^H have full column and row rank, respectively, with $m, p << n$. The unique stabilizing solution $X = X^H$ which exists under certain assumptions is positive semidefinite and makes the closed-loop matrix $A - BB^H X$ stable. Even so A is large and sparse, the solution X will still be a dense matrix in general. However, the above assumptions on B and C often imply that the sought-after solution X will have a low numerical rank (that is, its rank is $<< n$). This allows for the construction of iterative methods that approximate X with a series of low-rank matrices X_j stored in low-rank factored form. That is, the Hermitian low-rank approximations X_j to X are of the form $X_j = Z_j Y_j Z_j^H$, where Z_j is an $n \times kj$ matrix with only few columns and Y_j is a small square $kj \times kj$ Hermitian matrix.

We will first give an overview of methods that produce such a low-rank approximation. Afterward, we will delve into projection-type methods, which reduce the large-scale Riccati equation to a smaller one by projecting it onto specific block rational Krylov subspaces. These subspaces are spanned by blocks of the form $(A^H + \sigma_j I)C^H$, where σ_j are chosen shifts. The smaller, projected Riccati equation is then solved for the matrix Y_j which contributes to constructing the low-rank approximation X_j. We propose a new algorithm that, unlike traditional approaches, does not necessarily use an orthogonal projection. Our approach constructs projections directly from the matrices generated in the block rational Arnoldi decomposition of the block rational Krylov subspace. With this algorithm, the low-rank approximations X_j and the residual norm $||R(X_j)||_F$ can be computed quickly and efficiently. Finally, we will demonstrate the effectiveness of this method through numerical examples.

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