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Optimizing Rayleigh quotient with Hermitian and symmetric constraints

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Let $G, H \in \mathbb{C}^{n,n}$ be Hermitian and $S \in \mathbb{C}^{n,n}$ be a symmetric matrix. We consider the problem of maximizing the Rayleigh quotient of G with respect to constraints involving symmetric matrix S and Hermitian matrix H. More precisely, we compute

$$m(G,H,S) := \sup\left\{\frac{v^*Gv}{v^*v} :\ v \in \mathbb{C}^n \setminus \{0\},\, v^TSv = 0,\, v^*Hv = 0\right\}, \quad (1)$$

$$tag\mathrm{P} \quad (2)$$

where T and $*$ denote respectively the transpose and the conjugate transpose of a matrix or a vector.

Such problems occur in stability analysis of uncertain systems and in the eigenvalue perturbation theory of matrices and matrix polynomials [1, 2, 3]. A particular case of problem $(\mathcal{P})$ with only symmetric constraint (i.e., when $H = 0$) is used to characterize the μ-value of the matrix under skew-symmetric perturbations [3].

An explicit computable formula was obtained for $m(G, S)$ in [3, Theorem 6.3] and given by

m(G,S)=inf$_{t\in[0,\infty)}$ $\lambda_2\left(\begin{bmatrix} G & t\overline{S} \\ tS & \overline{G}\end{bmatrix}\right)$,

where $\lambda_2(A)$ stands for the second largest eigenvalue of a Hermitian matrix A. The case where only Hermitian constraint (i.e., when $S = 0$) was also considered and an explicit computable formula was obtained for $m(G, H)$ in [3, Theorem 6.2] and given by

m(G,H)=inf$_{t\in\mathbb{R}}$ $\lambda_{max}(G + tH)$,

where λ_{max} stands for the largest eigenvalue. However, the solution to the problem $(\mathcal{P})$ is still not known.

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