

Linear $\mathcal{L}$ -structured matrix equations

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We study the problem of constructing solutions [1, 2] $X \in \mathbb{R}^{\alpha \times \beta}$ of a linear matrix equation

$$LX = F, F \in \mathbb{R}^{\gamma \times \delta}, (1)$$

where $L : \mathbb{R}^{\alpha \times \beta} \rightarrow \mathbb{R}^{\gamma \times \delta}$ is a linear bounded matrix functional. A partial case of the matrix algebraic equation (1) is the well-known Sylvester and Lyapunov equations [1, 3, 4]. In the article [5], the definition of several partial cases was introduced for a matrix algebraic equation (1) with a $\mathcal{L}$ -structure

$$\mathcal{L}X = F, (2)$$

which determines the linear relationship between the elements of the solution of the matrix algebraic equation. In particular, the $\mathcal{L}$ -structure defines symmetric, skew-symmetric, diagonal matrices, as well as quaternions. Thus, we obtain the problem of finding solutions of a linear matrix equation (1) with a $\mathcal{L}$ -structure defined by a linear bounded matrix functional

$$\mathcal{L} : \mathbb{R}^{\alpha \times \beta} \rightarrow \mathbb{R}^{\lambda \times \mu};$$

here $F \in \mathbb{R}^{\lambda \times \mu}$ is a known matrix. In partial case, the $\mathcal{L}$ -structure defines magic squares [6], Hilbert, Hankel and Toeplitz matrices [7], Hermitian [8], symmetric and skew-symmetric matrices, as well as quaternions and biquaternions [9, 10].

References

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