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# Parametric interpolation for nonlinear dynamical systems using barycentric rational approximation of matrix-valued functions

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Parametrized nonlinear dynamical systems often arise upon discretizing parametrized partial differential equations (PDEs). Typically, the former tends to be of a large scale, i.e., the number of state variables could be significantly high (the state vector is denoted with x(t, p), p is the vector of parameters, the nonlinearity is f(x(t, p), p), while the matrices corresponding to the linear part are E(p), A(p), and B(p)). Solving it repeatedly at different parameter samples p (e.g., in the context of optimization and/or uncertainty quantification) tends to be an expensive pursuit. To address this, a reduced-order model (ROM) of the same structure can be obtained using well-known approaches such as proper orthogonal decomposition (POD), reduced basis method (RBM), and balanced truncation (BT). The number of equations in this formulation is usually far lesser compared to the full-order model, which makes it amenable to rapid, real-time evaluations, thereby circumventing expensive simulations associated with large-scale system evaluation.

In the context of the POD or RBM, snapshots of the solution vectors x(t, p), obtained at different time instances and different parameter samples are used to obtain a suitable projection matrix V which can be used to obtain the ROM via Galerkin projection.
A key implicit assumption in defining the reduced quantities thus obtained is that the matrices E(p), A(p), B(p), f(x(t, p), p) have an affine parameter dependence. While the affine parametric dependence assumption indeed holds in several applications, there are many cases where it does not hold. That is, it is impossible to know, a priori, the parameter affine form. In such scenarios, there is a need for a cheap and effective approximation technique to learn a function mapping the parameter p to the reduced matrices.

Pre-existing works that tackle the issue of non-affine parametrized matrices and/or vectors are only a few; in [Negri et al. '15] the authors introduced the matrix discrete empirical interpolation method (MDEIM) which leverages DEIM [Chaturantabut/Sorensen '10] to learn the mapping from the parameter to the reduced system matrix. However, this method needs data/entries of E(p) evaluated at a sparse set of locations. In the work [Degroote et al. '10], the authors propose an entry-wise interpolation of E(p) using splines. The same work also introduces a different technique of interpolating the reduced matrices over a Riemannian manifold for better approximation.

Recently, [Pelling et al. '24] exposed an interesting application of the classic Loewner approach for rational approximation in the context of parametric linear time-invariant (pLTI) systems. The authors utilize the snapshots of parametrized system matrices and the linear fractional transform to obtain ROMs of pLTIs. An intermediate step involved in the approach proposed in [Pelling et al. '24] is the use of the univariate Loewner approximation method in [Mayo/Antoulas '07] to interpolate large-scale system matrices E(p), A(p), and B(p). In our work, we rely on the observation of [Pelling et al. '24], but instead use it to learn a map from the parameter p to the reduced system matrices obtained via Galerkin projection. We do this by utilizing extensions of the Loewner framework for multiple parameters [Ionita/Antoulas '14], that rely on multivariate barycentric forms. Challenges that occur in this process come from dealing with a higher number of parameters (in the vector p) and with the non-scalar format of the data. We deal with these by accommodating the generalized barycentric forms to the matrix format (using barycentric formulas with matrix-valued weights as in [G./Guettel '21]), as well as allowing an adaptive choice of interpolation points (parameter values). Several numerical examples attest to the practical applicability of the proposed method.

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