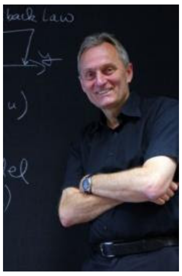

Contribution ID: **22** Type: **Talk**

Zeros, poles and system equivalence of time-delay systems

Tuesday 27 May 2025 12:00 (30 minutes)

\documentclass[11pt]{article}
\usepackage{amsmath,amssymb,amsfonts}
\usepackage{float,epsfig}
\textheight 8.9in
\textwidth 5.6in
\topmargin -0.5in
\newcommand{\se}{\setcounter{equation}{0}}
\renewcommand{\baselinestretch}{1.2}

\begin{document}

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\textbackslash{}title\{Zeros, poles and system equivalence of time-delay systems \}
\textbackslash{}author\{  \{\textbackslash{}bf Rafikul Alam\} and Jibrail Ali\textbackslash{}\textbackslas
    Department of Mathematics\textbackslash{}\textbackslash{} Indian Institute of Technology
    Guwahati \textbackslash{}\textbackslash{}Guwahati-781039, INDIA \}
\textbackslash{}date\{\}
\textbackslash{}maketitle \textbackslash{}thispagestyle\{empty\}
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\noindent
{\bf Extended abstract:} Consider the linear time-invariant time-delay system (TDS)
\begin{eqnarray} *\frac{\mathrm{d}\mathbf{x}(t)}{\mathrm{d}t} &=& A_0 \mathbf{x}(t) + \sum^{N_1}*{j=1} A_j \mathbf{x}(t - \tau_j) + \sum^{N_2}*{j=1} B_j \mathbf{u}(t- t_j), \\ \mathbf{y}(t) &=& \sum^{N_3}*{j=1} C_j \mathbf{x}(t- s_j) + \sum^{N_4}*{j=1} D_j \mathbf{u}(t-h_j), \end{eqnarray}*
where $\mathbf{x}(t) \in \mathbb{C}^r$ and $\mathbf{u}(t) \in \mathbb{C}^n$ are state and control vectors, respectively, at a time t, $(A_i, B_i, C_i, D_i) \in \mathbb{C}^{r\times r} \times \mathbb{C}^{r\times n} \times \mathbb{C}^{m\times r} \times \mathbb{C}^{m\times n}$ and (τ_i, t_i, s_i, h_i) are time-delay parameters. A system matrix $\mathbf{S}(\lambda)$ and the transfer function $\mathbf{M}(\lambda)$ of the TDS are given by
$\mathbf{S}(\lambda) := \left[\begin{array}{c|c} A(\lambda) & B(\lambda) \\ \hline -C(\lambda) & D(\lambda)\end{array}\right]$ and $\mathbf{M}(\lambda) := [D(\lambda)+C(\lambda)A(\lambda)^{-1}B(\lambda)], where$ A(\lambda) := \lambda I_r - A_0 - \sum^{N_1}*{j=1} A_j e^{-\lambda \tau_j}, B(\lambda) := \sum^{N_2}*{j=1} B_j e^{-\lambda t_j},C(\lambda) := \sum^{N_3}*{j=1} C_j e^{-\lambda s_j}and D(\lambda) := \sum^{N_4}*{j=1} D_j e^{-\lambda h_j}$areentirematrix-valuedfunctions.$

Our main aim is two-fold. First, to study the canonical forms of $\mathbf{S}(\lambda)$ and $\mathbf{M}(\lambda)$ so as to analyze zeros and poles of the TDS. Second, to investigate system equivalence, that is, if $\mathbf{S}$*1(\lambda)and\mathbf{S}_2(\lambda)*$aresystemmatricesoftime-delaysystems, th$

We show that $\mathbf{M}(\lambda)$ admits a Smith-McMillan form $\Sigma{\mathbf{M}}(\lambda)$ given by

$$\Sigma_{\mathbf{M}}(\lambda) = \left[\begin{array}{ccc|c} \phi_1(\lambda)/\psi_1(\lambda) & & & \\ & \ddots & & \\ & & \phi_p(\lambda)/\psi_p(\lambda) & \\ \hline & & & 0_{m-p\times n-p}\end{array}\right], \qquad (1)$$

where \phi_1, \ldots, \phi_p and \psi_1, \ldots, \psi_p are entire functions, \phi_j and \psi_j are relatively prime. Further, \phi_j divides \phi_{j + 1} and \psi_{j + 1} divides \psi_j for $j = 1 : p - 1$. Furthermore, \phi_1, \cdots, \phi_p and \psi_1, \ldots, \psi_p are given by $\phi_j(\lambda) = \prod_{\ell=1}^{\infty}(\lambda - \lambda_\ell)^{\partial_j(\lambda_\ell)} u_{j\ell}(\lambda)$ and $\psi_j(\lambda) = \prod_{\ell=1}^{\infty}(\lambda - \mu_\ell)^{\delta_j(\mu_\ell)} v_{j\ell}(\lambda)$, *where* u_{j\ell}, v_{j\ell} *areentirefunctionswithnozerosin*\mathbb
*negativeintegersfor*j=1:p*and*\ell \in \mathbb{N}.

We also show that \mathbf{M}(\lambda) admits a right coprime matrix-fraction description (MFD), that is, there exist entire matrix-valued functions N(\lambda) and D(\lambda) such that N(\lambda) and D(\lambda) are right coprime, D(\lambda) is regular, and M(λ) = $N(\lambda)D(\lambda)^{-1}$.*Further,*\sigma_{\mathbb{C}}(\mathbf
= \sigma_{\mathbb{C}}(N)*and*\wp_{\mathbb{C}}(\mathbf{M}) = \sigma_{\mathbb{C}}(D),*where*\sigma_{\mathbb{C}}(X)*isthespe*
*valuedfunction*X(\lambda).

For system matrices, we show that S$_1(\lambda) \sim_{rse}$ $\mathbf{S}_2(\lambda) \Longleftrightarrow \mathbf{S}_1(\lambda) \sim_{fse} \mathbf{S}_2(\lambda)$.*Further, if*\mathbf{S}*1(\lambda)and\mathbf{S*
\sim{rse} \mathbf{S}*2(\lambda) \Longleftrightarrow \mathbf{S}_1(\lambda) \text{ and } \mathbf{S}_2(\lambda)havethesametran*
McMillanformof\mathbf{M}(\lambda)*asgivenabove, thenweshowthatto*
$S_A(\lambda) = I_{r-p} \oplus \mathrm{diag}(\psi_p(\lambda), \psi_{p-1}(\lambda), \psi_1(\lambda))$
$S_{\mathbf{S}}(\lambda) = I_r \oplus \mathrm{diag}(\phi_1(\lambda), \ldots, \phi_p(\lambda)) \oplus 0_{m-p,n-p}$ are the Smith forms of A(\lambda) and \mathbf{S}(\lambda), respectively. Hence we show that \sigma_{\mathbb{C}}(\mathbf{M}) = \sigma_{\mathbb{C}}(\mathbf{S}) and \wp_{\mathbb{C}}(\mathbf{M}) = \sigma_{\mathbb{C}}(A).

vspace2ex
noindent **References:**

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Session Classification: Talks