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Symplectic Integrators for Differential Riccati Equations

Let $A, S, X_0 \in \mathbb{R}^{n\times n}$ and $B \in \mathbb{R}^{n\times b}$ be. Moreover assume that S, X_0 are symmetric positive semidefinite.
We consider
\begin{align}
\dot{X}(t) &= A^T X (t) + X(t) A - X(t) S X(t) + C^T C, \\\\
X(0)&=X_0.
\end{align}
It is well known that the solution X can be obtained from the Hamiltonian system
\begin{align}
\begin{bmatrix} \dot{U}(t) \\\\ \dot{V}(t) \end{bmatrix}
&=
-H \begin{bmatrix} U(t) \\\\ V(t) \end{bmatrix}
=
-\begin{bmatrix} A & -S \\\\ -C^T C & -A^T \end{bmatrix}
\begin{bmatrix} U(t) \\\\ V(t) \end{bmatrix}, \\\\
\begin{bmatrix} U(0) \\\\ V(0) \end{bmatrix}
&=
\begin{bmatrix} I_n \\\\ X_0 \end{bmatrix},
\end{align}
by the formula $X(t) = V(t)U(t)^{-1}$.
The solution of the Hamiltonian system can be expressed in terms of the matrix exponential of the Hamiltonian $-H$, which is symplectic.
This motivates to approximate the flow by numerical schemes, which uses symplectic transformations.

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