

# Analysis of bubble dynamics using data-driven dynamical systems and machine learning

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The formation and oscillation of bubbles is important in cavitation related to turbomachinery, and in biomedical applications, such as contrast-enhanced ultrasound imaging and drug delivery for cancer treatment. There is an extensive literature on the modeling and analysis of bubble oscillations in these settings, allowing for detailed simulations from first principles. However, there are still many open questions that may benefit from machine learning (ML). In this research, we apply data-driven and ML methods to analyzing and controlling the nonlinear dynamics of bubble oscillations. In this context, the Rayleigh-Plesset equation (RPE) is a central object of study [1]. It exhibits richly-structured chaotic solutions when describing an acoustically-driven bubble for certain parameter values [2]. Nonspherical shape modes - which are important for enhancing ultrasound imaging and promoting drug delivery - can be overlaid as perturbations to the basic spherical mode [3, 4, 5], leading to a dynamical system of much higher dimension. Recently, experimental studies of bubble shape modes evolving in acoustic fields have captured large amounts of high-quality time series data [6, 7, 8, 9]. We are therefore interested in discovering reduced-order models of microbubble dynamics from raw experimental data and comparing these to data-driven analyses of first-principle, physics-based models. Additionally, we want to apply our data-driven model to develop a framework for nonlinear control [10] of both individual bubbles and bubbly flows using acoustic forcing. To this end, we have developed a deep neural network (DNN) to forecast time series previously generated numerically from the RPE. We intend to train this on experimental time series and use it to predict the dynamic response of bubbles to changes in acoustic forcing. We are also exploring the Singular Value Decomposition (SVD) of Hankel matrices built from these time series to identify a Koopman embedding of the RPE when acoustically-driven. This Koopman embedding provides a coordinate system wherein the nonlinear dynamics of bubble oscillations becomes linear, allowing the application of tools from classical control theory.

## References

- [1] Christopher Earls Brennen. Cavitation and Bubble Dynamics. Cambridge University Press, Cambridge, 2013.
- [2] Werner Lauterborn and Engelbert Suchla. Bifurcation Superstructure in a Model of Acoustic Turbulence. *Physical Review Letters*, 53(24):2304-2307, December 1984. Publisher: American Physical Society.
- [3] Michael Calvisi, Olgert Lindau, John Blake, and Andrew Szeri. Shape Stability and Violent Collapse of Microbubbles in Acoustic Traveling Waves. *Physics of Fluids*, 19, April 2007.
- [4] M. S. Plesset. On the Stability of Fluid Flows with Spherical Symmetry. *Journal of Applied Physics*, 25(1):96-98, January 1954. Publisher: American Institute of Physics.
- [5] Matthieu Guedra and Claude Inserra. Bubble shape oscillations of finite amplitude. *Journal of Fluid Mechanics*, 857:681-703, 2018. Edition: 2018/10/25 Publisher: Cambridge University Press.
- [6] Sarah Cleve, Matthieu Guedra, Claude Inserra, Cyril Mauger, and Philippe Blanc-Benon. Surface modes with controlled axisymmetry triggered by bubble coalescence in a high-amplitude acoustic field. *Physical Review E*, 98, September 2018.
- [7] M. Guedra, C. Inserra, B. Gilles, and C. Mauger. Periodic onset of bubble shape instabilities and their influence on the spherical mode. In 2016 IEEE International Ultrasonics Symposium (IUS), pages 1-4, September 2016. Journal Abbreviation: 2016 IEEE International Ultrasonics Symposium (IUS).
- [8] Matthieu Guedra, Sarah Cleve, Cyril Mauger, Philippe Blanc-Benon, and Claude Inserra. Dynamics of nonspherical microbubble oscillations above instability threshold. *Physical Review E*, 96(6):063104, December 2017. Publisher: American Physical Society.
- [9] Matthieu Guedra, Sarah Cleve, Cyril Mauger, Claude Inserra, and Philippe Blanc-Benon. Time-resolved dynamics of micrometer-sized bubbles undergoing shape oscillations. *The Journal of the Acoustical Society of America*, 141:3736-3736, May 2017.
- [10] Joshua Proctor, Steven Brunton, and J. Kutz. Generalizing Koopman Theory to Allow for Inputs and Control. *SIAM Journal on Applied Dynamical Systems*, 17, February 2016.

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